

Optimal model through identified frequencies of a masonry building structure with wooden floors

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Abstract—The paper presents the analysis of an important historical building: the Saint James Theater in the city of Corfù (Greece) actually used as the Municipality House.

The building, located in the center of the city, is made of carves stones and is characterized by a stocky shape and by the presence of wooden floors.

The study deals with the structural identification of such structure through the analysis of its ambient vibrations recorded by means of accelerometers with high accuracy. A full dynamic testing was developed using ambient vibrations to identify the main modal parameters and to make a non-destructive characterization of this building.

The results of these dynamic tests are compared with the modal analysis of a complex finite element (FE) simulation of the structure. This analysis may present several problems and uncertainties for this stocky building. Due to the presence of wooden floors, the local modes can be highly excited and, as a consequence, the evaluation of the structural modal parameters presents some difficulties.

Keywords—Historical building, Non-destructive tests, Operational modal analysis, Dynamic analysis, FEM analysis.

I. INTRODUCTION

THE construction of Saint James building was started in 1663 but it stopped for a period, probably due to financial problems and continued in 1687. The building was completed in 1693. Initially, the structure was built as lodge for the nobles and was known with the name “Loggia”. Only in 1720 it was renamed as Saint James, like the close catholic Cathedral, and was converted in a theatre. At the end of 1903 it was transformed into a City Hall when the insertion at the front part of the building was dismantled and one more floor in the central part was built.

This masonry historical building has a rectangular plant

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absolutely symmetric, with dimensions 24.75 m (Fig. 1) and 14 m (Fig. 2). The building has a maximum height of 19 m. Five domes arched windows in a row at the two main sides and two symmetric rectangle windows in each narrow side characterize the structure. In the main façade the central arched window is modified to build the main entrance to the building from Dimarchiou square.



Fig. 1 Main façade. South façade. Dimarchiou square (Saint James building Corfù Greece)

This structure has a semi-basement for the South façade. Fig. 2 shows a South-West lateral façade where it is possible to identify this semi-basement for the South façade and the first basement in the North façade.



Fig.2 Lateral façade. North-East (Left). South-West (Right) (Saint James building Corfù Greece)

Inside the building has three different levels of wooden floors; the third was built some year after the initial building. Some load-bearing walls ensure the stability of this building in its two main orthogonal axes (Fig. 3). The described configuration underlines that the building is extremely stocky, and consequently some difficulties are reasonably expected in the dynamic identification analysis. Moreover, the presence of wooden floors does not guarantee about the rigid behavior of the floors and so the dynamic response of the structure may be governed by “local” modes instead of the “global” ones. Therefore, to overcome this problem an *ad hoc* procedure is here proposed for the experimental set-up and for the dynamic identification analysis, that has the aim of verifying the influence of local modes on the dynamic response of the structure.



Fig. 3. Longitudinal section. North view. Wooden floor location (Saint James building Corfu Greece)

II. NUMERICAL MODEL

The numerical analysis of different historical masonry constructions has been studied by many authors [1-4] that have explained the complexity to model the behavior of masonry construction under static and dynamic loads. To obtain a homogeneous numerical model, the technique of modal identification is a useful procedure for this structural typology [5-26] even if it fits better for slender reinforced concrete structures [27-29].

Here a detailed 3D numerical model was developed to simulate the structural behavior of Saint James building (Fig. 4). SAP2000 commercial software [30] was used considering an initial linear behavior for the masonry and wooden materials. This software uses the Finite Element Method to simulate the structural behavior. The connection between the soil and the constructions has been considered rigid, and the connection between the semi-basement and the perimeter soil has been considered rigid too. 15259 shell elements have been utilized to simulate the different structural elements: resisting walls, cover and floors. 15139 nodes and 162 frames elements have been utilized to simulate the wooden beams supporting the top cover of the building (Fig.5). The hypothesis of thick-shell elements has been considered for all the surface elements, defined using 4 nodes and 6 degrees of freedom for each node. Thickness between 0.3 m and 1.05 m has been considered to simulate the real structural elements. The wooden floor

elements have been modeled using a thickness of 0.25 m that can be considered as a mean value chosen in order to simulate the wooden beams and the wooden deck platform.

Interesting approaches for the mechanical characterization of the structural materials (density and elastic properties) have been studied in literature [31-32], whereas problems connected either to the possibility of employing innovative materials or to model and test suitable technical interventions for improving these properties may be found in [33-37]. Moreover, recent possible progresses contained in [38] suggest new approaches. Here, a non-destructive approach based on the comparison between the identified experimental modal parameters and the FE model is proposed. No specific tests have been initially carried on to obtain the main characteristics of the structural materials: masonry and wooden. Table 1 shows some initial values for these materials obtained from [1].

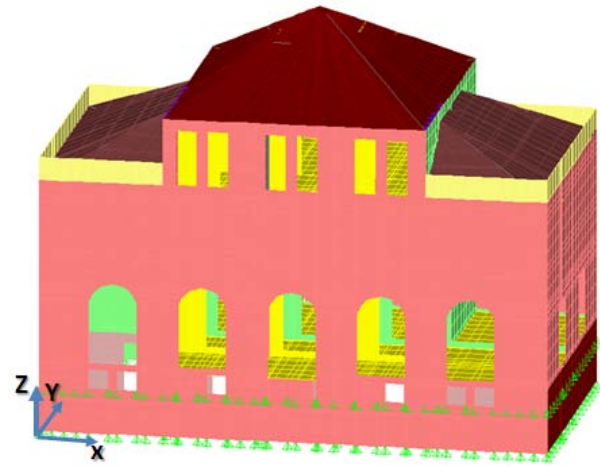


Fig. 4. General view of the 3D numerical model.

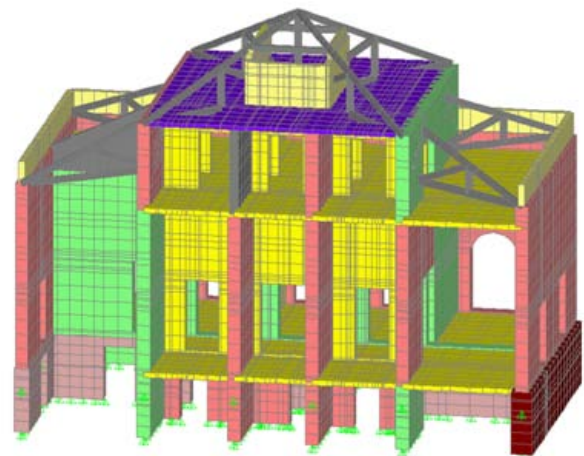


Fig. 5 3D Numerical model. Longitudinal section. South view.

Table 1. Characteristics of the materials

	Elastic modulus (MPa)	Density (kg/m ³)
Masonry	5883	1800
Wooden	14709	800

To simulate the live load on each floor (levels 1 and 2) a constant load of 2 kN/m^2 has been considered on the floor elements.

An initial static analysis of the main walls of this building indicates that the maximum compression stresses reached at one of the surface of the shell elements and due to the self-weight and the live loads (Fig. 6) are lower than 1 MPa , completely admissible for a conventional masonry. This analysis allows to state that the maximum tension stresses reached at the other surface of the shell elements are lower than 1 MPa , a value completely admissible for a conventional masonry (Fig. 7).

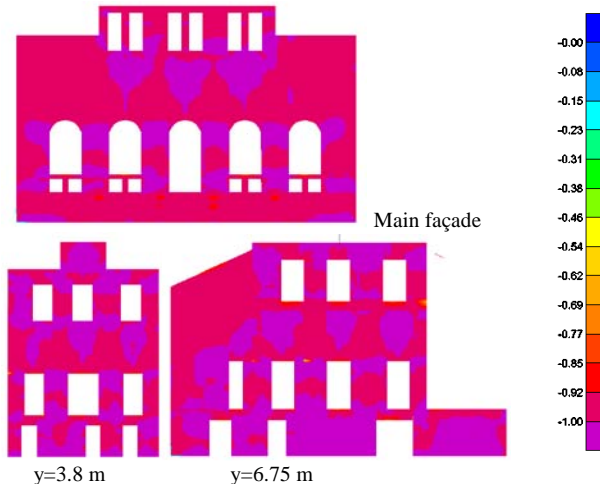


Fig. 6. Maximum normal compression stresses in the main walls. (N/mm^2) due to the self-weight and the live loads

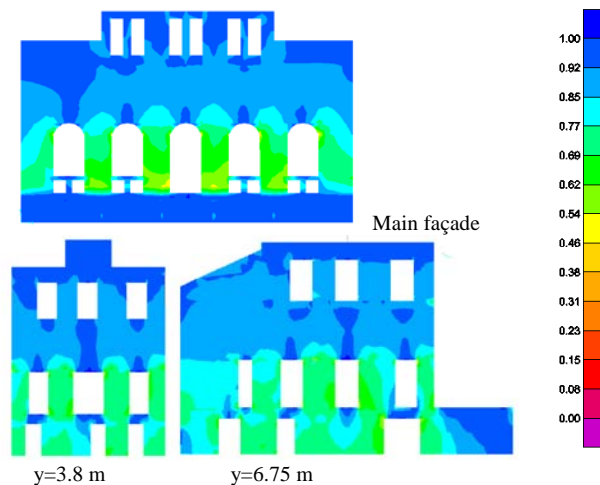


Fig 7. Maximum normal tension stresses in the main walls. (N/mm^2) due to self-weight and the live loads .

For this preliminary analysis the stresses on the different floors under the self-weight and the live load of 2 kN/m^2 have been considered. Figs. 8 and 9 present, respectively, the maximum and minimum normal stresses on each floor (level 1 at $h=3 \text{ m}$ and level 2 at $h=10.9 \text{ m}$). The results shown here are lower than the maximum admissible values for a conventional

wooden structure well preserved.

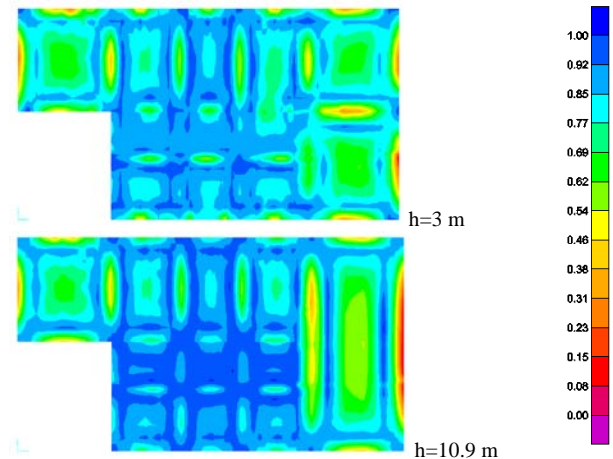


Fig. 8. Maximum normal tension stresses in wooden floors. (N/mm^2) due to self-weight and the live loads.

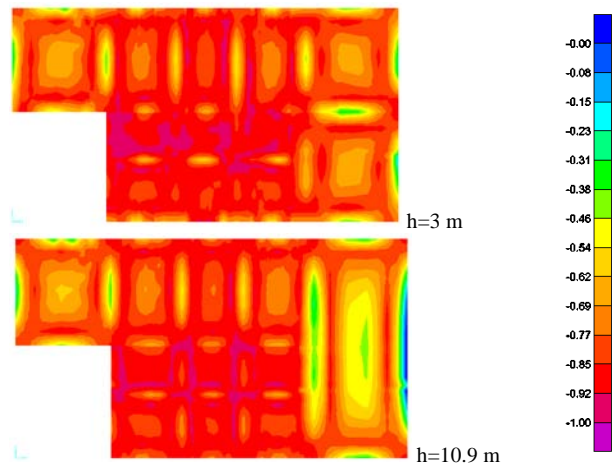


Fig. 9. Maximum normal compression stresses in wooden floors. (N/mm^2) due to self-weight and the live loads

A modal analysis has been developed to obtain the identification of the main frequencies of the building. Fig. 10 presents a visual identification of these modes. Table 2 indicates the total mobilized mass and the numerical values of the main frequencies.

III. THE EXPERIMENTAL SETUP FOR THE DYNAMIC TESTS

The experimental monitoring phase has been performed on 10th and 11th of July 2013. The monitoring system consists of several elements properly connected: 18 seismic accelerometers ICP PCB 393B31, the data acquisition system or DAQs positioned at each level monitored; the laptop with acquisition software; the cables that connect all elements to each other.

Nine points of the building have been monitored by installing in each point two accelerometers on appropriate

rectangular blocks (see Fig. 11) in order to ensure the orthogonality of the couple of sensors. The monitoring system has been positioned at different levels and in different rooms, according to the orthogonal directions x and y defined by the orthogonal main façades.

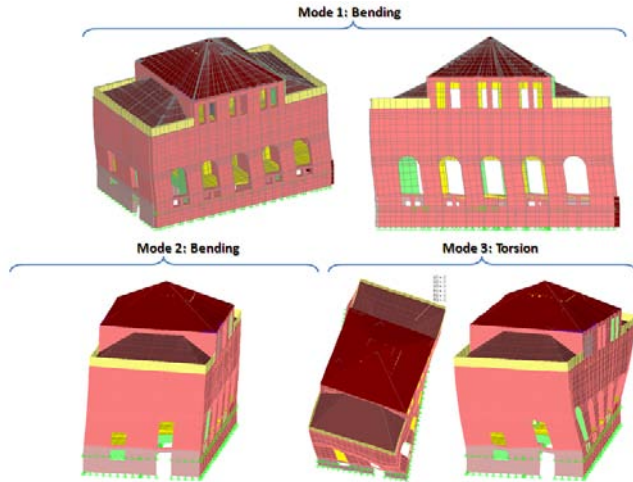


Fig. 10. Modal analysis results.

Table 2. Identified periods and frequencies of the FE model

Mode	Period (s)	Frequency (Hz)	Sum UX	Sum UY	Sum UZ	Sum RX	Sum RY	Sum RZ
1	0.164	6.09	0.71	0	0	0	0.3	0.12
2	0.131	7.64	0.71	0.65	0	0.6	0.3	0.41
3	0.108	9.24	0.71	0.66	0	0.61	0.3	0.64
4	0.095	10.52	0.71	0.66	0	0.61	0.3	0.65
5	0.094	10.66	0.71	0.66	0	0.61	0.3	0.65

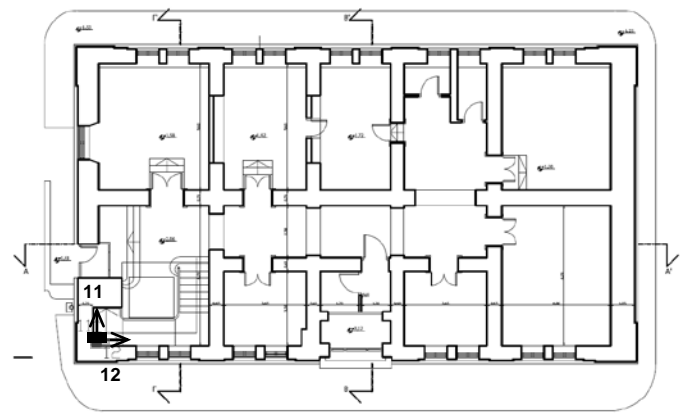
Fig. 11 shows the biaxial configuration of the accelerometers in two different positions on the walls prepared to register longitudinal and transversal vibrations of the building.

The monitored points are sketched in Fig.12 referred to the semi-basement level (a), to the first level (b) and to the second level (c) of the structure. In Fig. 12 the arrows indicate the acquisition direction of each accelerometer. Each data acquisition record was carried out by 10 minutes recordings with a sampling frequency of 1024 Hz, which has been subsequently decimated by a factor equal to 4 to have a sampling frequency of 256 Hz for the subsequent analysis.

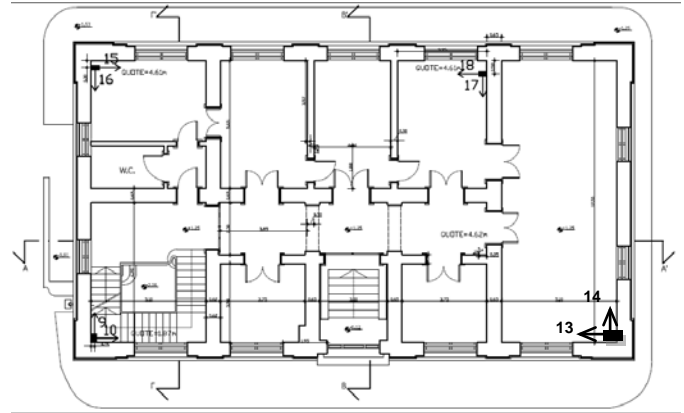


Fig. 11. Seismic accelerometers located in different positions on the building.

(a)



(b)



(c)

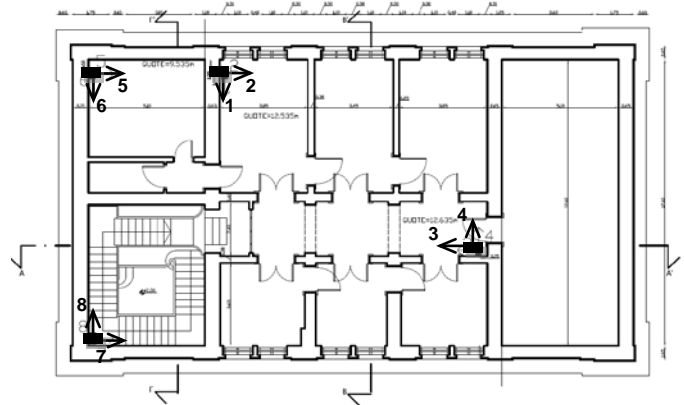


Fig. 12. The plan views of the Saint James building: a) semi-basement level; b) first level; c) second level.

IV. THE DATA PROCESSING

The recorded data have been processed to identify the natural frequencies and the modal shapes of the Saint James building by using the specialized Artemis software [39]. At this proposal, a spatial model of the building and of the position of the accelerometers has been created as shown in Fig.13.

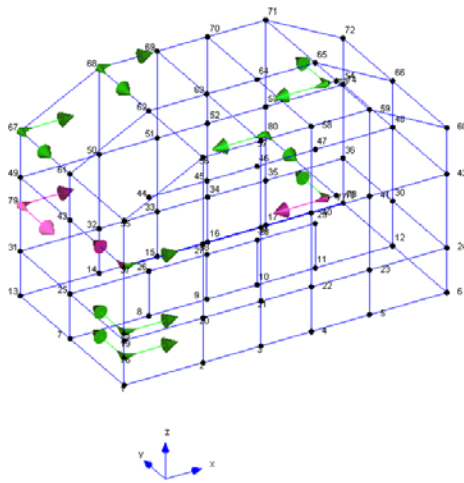
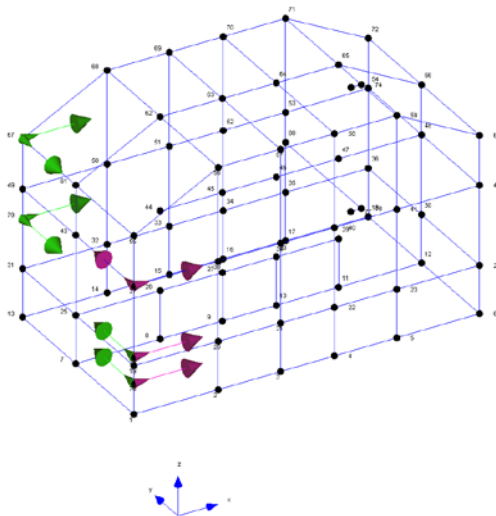


Fig. 13. Artemis model of the Saint James building; the arrows indicate the acquisition direction of the 18 accelerometers applied to the structure.

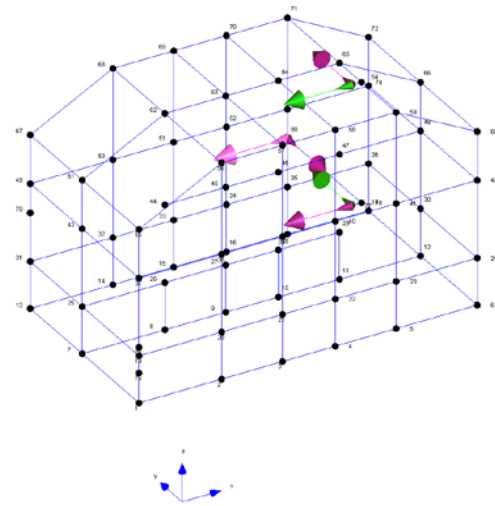
The acquired signals in environmental conditions have a very low amplitude, due to the very squat profile of the

structure (as shown in [15,21]). For this reason, as demonstrated in [40], only the Stochastic Subspace Identification (SSI) method [41-42] essentially based on fitting to dynamic response discrete-time data and more robust for low significant data, has been used for performing the Operational Modal Analysis (OMA) in this case.

Moreover, in order to verify the contribution of local modes on the dynamic response of the examined building, the following strategy has been proposed: the data have been analyzed taking into account only the accelerometers positioned on the same wall. So, two groups of accelerometers have been considered; the first composed by the accelerometers (named by the numbers as indicated in Fig.12) 5,6,7,8,9,10,11,12,15,16 related to the perimeter wall parallel to the y axis (Fig.13) on the left part of the building. The second composed by the accelerometers 3,4,13,14,17,18 related to an internal wall parallel to the same y axis on the right part of the building. In Fig. 14 the two groups considered are indicated directly on the Artemis model not considering the presence of the other sensors.



a)



b)

Figure 14: the two groups of accelerometers separately considered: a) first group; b) second group.

The aim of the procedure is to perform the identification analysis considering each wall alone (i.e. taking into account the data related to the accelerometers installed on the examined wall) and evaluating the natural frequencies. The main idea is that if the dynamic response of the building is governed by local modes, the natural frequencies estimated for each wall are expected to be different otherwise may be related to “global” modes. The SSI method has been applied to ten different acquisitions in such a way to have a statistical identification considering the possible modal parameters uncertainty [43]; a sample of the SSI application (first group

of accelerometers, test 1) with maximum order 100 is shown in Fig.15. The SSI diagram in Fig. 15 demonstrates that the peaks corresponding to the frequencies are not well highlighted, probably due to the very squat and fixed shape of the structure. For this reason, the damping evaluation [44] has not been considered in the following comparison. A certain number of frequencies may be identified; considering the repeatability of the frequencies all over the ten considered acquisitions, the frequencies repeated over the 50% of the tests have been considered for the following analysis.

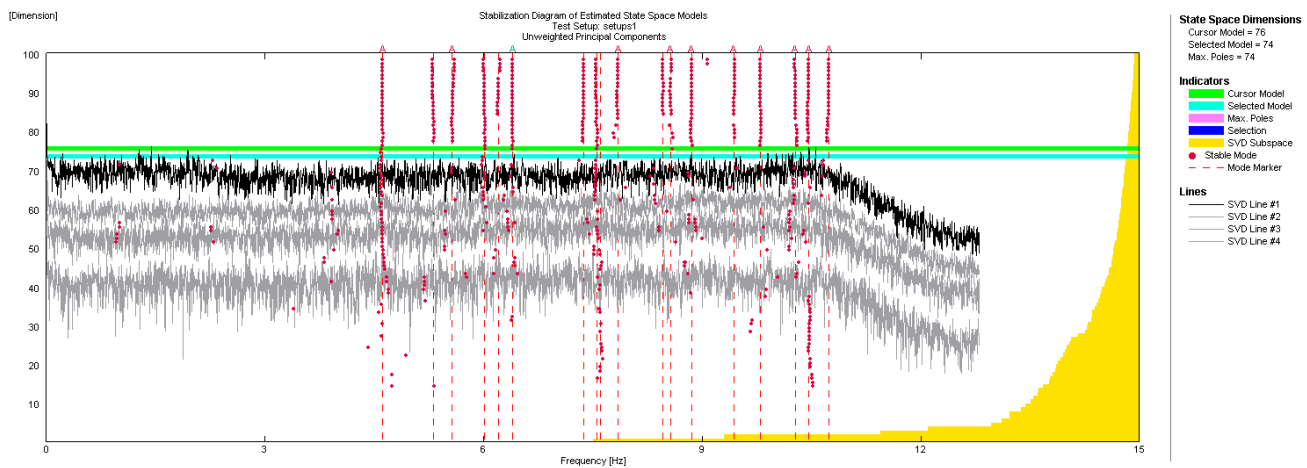


Fig. 15. Results of SSI methods for the first test of the first group of accelerometers.

Table 3 shows the average value on the ten tests and the standard deviation of the first five identified frequencies by SSI method for the two considered groups of accelerometers.

Table 3. Statistical characteristic of the first five identified frequencies with SSI method for the two groups on ten tests.

n. identified frequency	Average value for the first group [Hz]	Standard deviation for the first group	Average value for the second group [Hz]	Standard deviation for the second group
1	4.69	0.044	4.68	0.017
2	5.74	0.065	5.73	0.028
3	6.51	0.053	6.50	0.036
4	7.60	0.049	7.66	0.040
5	8.88	0.036	8.87	0.064

The results in Table 3 clearly demonstrate that the identified frequencies are very repeatable and stable for both groups showing a very low standard deviation on the 10 experimental tests. Moreover, the frequency values are practically coincident for the examined groups; consequently, such frequencies can be reasonably considered as related to “global” modes. This information is very important for a subsequent phase of model validation and updating.

A subsequent step of the research will be the comparison between the estimated frequencies and the ones evaluated performing the operation modal analysis considering all the accelerometers and the rigid floors hypothesis.

The actual differences between the identified frequencies and the numerical ones may be due to some hypothesis regarding the total mass of the building and the mechanical characteristics of the masonry, but the entities of such differences make confident of the updating procedure of the finite element model.

These initial values permit to affirm that the numerical model is less rigid than the real model. To obtain more realistic results for the numerical model, it is necessary to define the main parameters to homogenize the total masonry building only using two materials: masonry and wood.

I. CONCLUSIONS

In this work, an optimized FE model of a masonry building structure with wooden floors has been presented and discussed with the aim of comparing the numerical frequencies with the identified ones. The dynamical identification using accelerometers has been carried out estimating the first five frequencies of the structure and analyzing, with a local wall- to wall analysis, their global character; the identification accuracy has been guaranteed by considering several experimental data referred to consecutive acquisitions. In this sense, a statistical approach has been performed. This analysis will allow, in future researches, to obtain important information about the character of the mode shapes and also the correct values of the material properties.

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